The Social and Philosophical Consequences of Mathematization

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3 consequences of Mathematization

- Social
  - Social exclusion
- Epistemological
  - Transformation of the meaning of « explanation »; Algorithmes et automates cellulaires vs équations différentielles
- Ontological
  - Desubstantialization

Vanishing substances

- Mechanical cause of gravity
- Ether
- Concept of Mass
- Lorentz contraction
- Spin
- Algorithmes vs équations différentielles

La puissance heuristique des analogies: de la syntaxe à la sémantique

Planck

$$\rho = \frac{8\pi h v^3}{c^3} \frac{1}{e^{hv/kT} - 1}$$

Limits:

Wien

$$\rho = \frac{8\pi h\nu^3}{c^3} e^{-h\nu/kT}$$

Rayleigh-Jeans

$$\rho = \frac{8\pi\nu^2}{c^3} kT$$

Boltzmann's Principle

$$S = k \ln W \quad .$$

Entropy of radiation

$$S - S_0 = k \ln(V / V_0)^{E/h\nu}$$

Entropy of a gaz

$$S - S_0 = k \ln(V / V_0)^{n}.$$

formal Analogy implies:

$$E = nh\nu$$

Fluctuations (1904)

$$\overline{\varepsilon^2} = kT^2 \frac{\partial \overline{E}}{\partial T} \cdot$$

Fluctuations of Planck radiation (1909)

$$\overline{\varepsilon^2} = (h\nu\rho + \frac{c^3}{8\pi\nu^2}\rho^2)VdV \quad .$$

“the first term, if present alone, would yield a fluctuation of the radiation energy equal to that produced if the radiation consisted of point quanta of energy $h\nu$ **moving independently of each other”**

“the next stage in the development of theoretical physics will bring us a theory of light that can be understood as a kind of fusion of the wave and emission [read: particle] theories of light

Energy Fluctuations in a solid (1911)

$$\overline{\left[\frac{\varepsilon}{E}\right]^2} = \frac{1}{z_q} + \frac{1}{z_f}$$

$Z_q = \dfrac{E}{hv}$ = average number of photons in the solid

$Z_f = 3nN$ = total number of degrees of freedom in the solid

N = Avogadro number

one sees from this equation that <u>the system's relative energy fluctuations</u>, wh ich are produced by the irregular thermal motion, <u>result from two completely different causes</u>" [<u>the second term] i s the only fluctuation according [to classical mechanics</u> while the first term should be absent though it i s the one obtained from the] quantum hypothesis, according to which energy consists of quanta of magnitude $h\nu$, which change their location independently of each other

Fluctuations in the number of molecules

$$\overline{\left[\frac{\Delta_\nu}{n_\nu}\right]^2} = \frac{1}{n_\nu} + \frac{1}{z_\nu}$$

Δ_ν =average number of molécules n_ν in energy range ΔE

z_ν = number of phase cells in energy range ΔE

“the square of the average relative fluctuation of the molecules is composed additively of two terms. Only the first of these would be present if the molecules were independent of each other.”

Louis de Broglie, C.R. Nov. 1922

OPTIQUE. — *Sur les interférences et la théorie des quanta de lumière.* Note de M. **Louis de Broglie**, présentée par M. Deslandres.

Enfin, dans le cas *réel*, celui de la loi de Planck

$$E = \frac{8\pi h}{c^3}\nu^3 \frac{1}{e^{\frac{h\nu}{kT}} - 1} V\,d\nu,$$

on trouve, comme M. Einstein l'a montré au Congrès de Bruxelles en 1911,

$$\overline{\varepsilon^2} = h\nu E + \frac{c^3}{8\pi\nu^2\,d\nu}\frac{E^2}{V};$$

$\overline{\varepsilon^2}$ est donc la somme de ce qu'il serait : 1° si la radiation était purement ondulatoire; 2° si la radiation était entièrement divisée en quanta $h\nu$.

Au point de vue de la théorie des quanta de lumière, il paraît logique d'écrire la formule de Planck sous la forme suivante :

$$E = \frac{8\pi h}{c^3}\nu^3 e^{-\frac{h\nu}{kT}} V\,d\nu + \frac{8\pi h}{c^3}\nu^3 e^{-\frac{2h\nu}{kT}} V\,d\nu + \ldots$$
$$= \sum_{1}^{\infty}{}_{n} \frac{8\pi h}{c^3}\nu^3 e^{-\frac{nh\nu}{kT}} V\,d\nu = E_1 + E_2 + \ldots + E_n + \ldots,$$

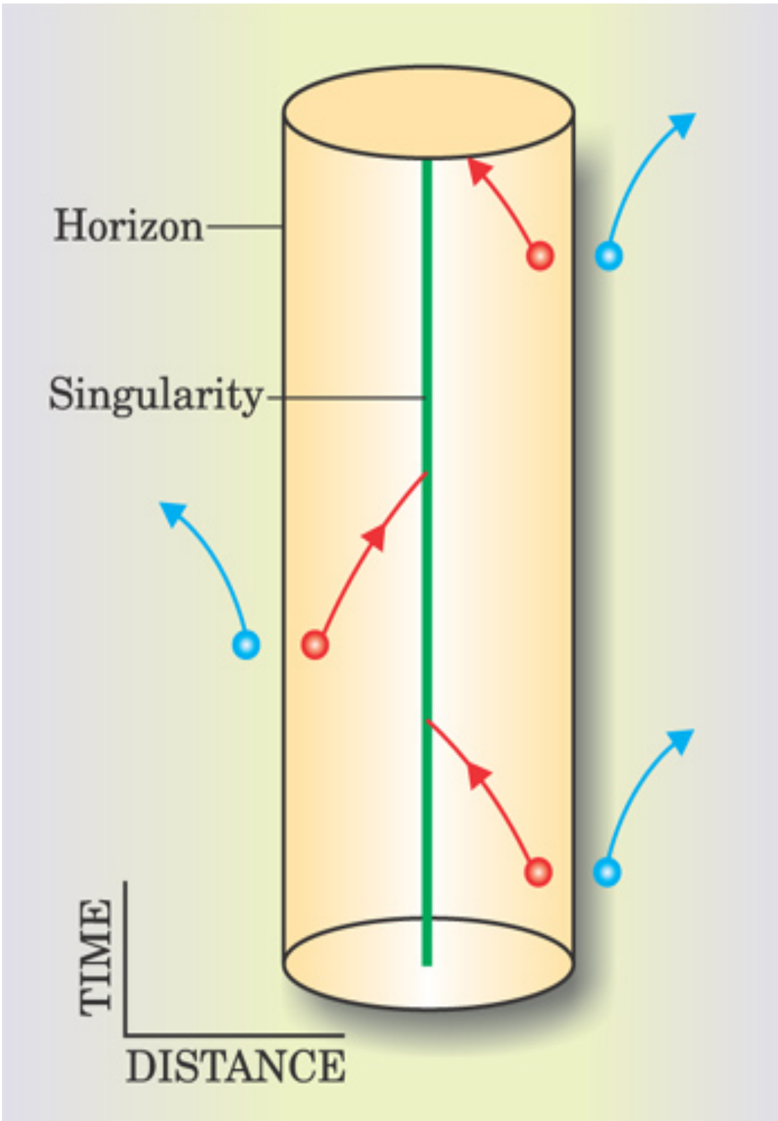
Le premier terme E_1 correspondrait à l'énergie divisée en quanta $h\nu$, le second E_2 à l'énergie divisée en quanta $2h\nu$ (molécules de lumière à 2^{at}), et ainsi de suite. La formule des fluctuations donne alors

$$\overline{\varepsilon^2} = h\nu E_1 + 2h\nu E_2 + 3h\nu E_3 + \ldots = \sum_{1}^{\infty}{}_{n}\, n\,h\nu E_n,$$

et cette formule est bien celle qui correspond à un « gaz de lumière » formé de molécules et d'atomes. Naturellement, cette nouvelle forme est identique au fond à celle d'Einstein en raison de l'identité facile à vérifier

$$\sum_{1}^{\infty}{}_{n} (n-1)h\nu E_n = \frac{c^3}{8\pi\nu^2\,d\nu}\frac{E^2}{V}.$$

Si l'on examine bien ces formules, on verra qu'elles ont la signification suivante : Au point de vue des quanta de lumière, les phénomènes d'interférences paraissent liés à l'existence d'agglomérations d'atomes de lumière dont les mouvements ne sont pas indépendants, sont cohérents. Dès lors, il est naturel de supposer que si la théorie des quanta de lumière parvient un


Horizon
Singularity
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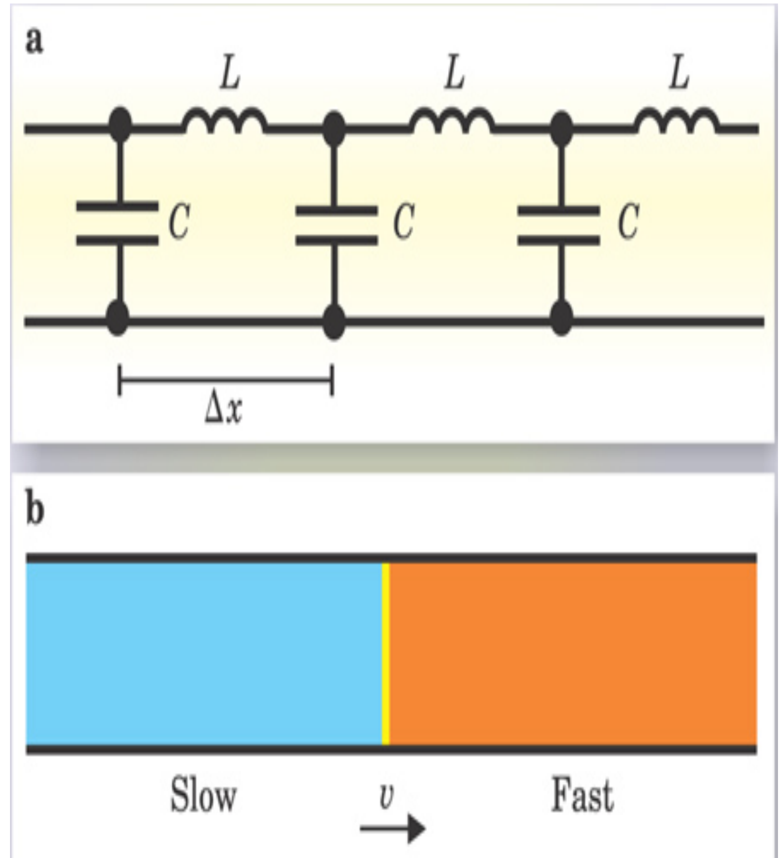
a
L
L
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b
Slow
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Fast

Hawking radiation in an electro-magnetic wave-guide?

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It is demonstrated that the propagation of electro-magnetic waves in an appropriately designed wave-guide is (for large wave-lengths) analogous to that within a curved space-time – such as around a black hole. As electro-magnetic radiation (e.g., micro-waves) can be controlled, amplified, and detected (with present-day technology) much easier than sound, for example, we propose a set-up for the experimental verification of the Hawking effect. Apart from experimentally testing this striking prediction, this would facilitate the investigation of the trans-Planckian problem.

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Introduction One of the major motivations behind the idea of black hole analogues ("dumb holes", see [1]) is the possibility of an experimental verification of the Hawking effect [2]. Apart from testing one of the most striking theoretical predictions of quantum field theory under the influence of external conditions, such an experiment would enable us to investigate the impact of ultra-high energy/momentum degrees of freedom (trans-Planckian problem) on the lowest-order Hawking effect and its higher-order corrections (with respect to the small ratio of Hawking temperature over Planck scale) by means of an analogue system. In view of the close relation between the Hawking effect and the concept of black hole entropy, these investigations are potentially relevant for the black hole information paradox etc.

The analogy between sound waves in moving fluids and scalar fields in curved space-times established in [1] can (in principle) be used to simulate a horizon in liquid Helium [3] or in Bose-Einstein condensates [4], for example (see also [5]). However, measuring the Hawking effect in those systems goes along with serious difficulties [6]. The main problem is the detection of sound waves corresponding to the realistically very low Hawking temperature.

On the other hand, electro-magnetic radiation is much easier to amplify and to detect with present-day technol- the propagating electro-magnetic waves

$$\delta z \ll \delta x \ll \Delta x, \Delta z \ll \Delta y \ll \lambda\,. \qquad (1)$$

In this limit, the wave-guide possesses a large slow-down and we can omit Maxwell's supplement $\dot{\boldsymbol{D}}$ in Oerstedt-Ampère's law $\boldsymbol{\nabla}\times\boldsymbol{H} = \boldsymbol{j} + \dot{\boldsymbol{D}} \leadsto \oint d\boldsymbol{r}\cdot\boldsymbol{H} = I + \frac{d}{dt}\int d\boldsymbol{S}\cdot\boldsymbol{D}$ in the upper region of the wave-guide (i.e., the surface integral over $S = \Delta x \times \Delta z$). The conditions (1) also ensure that the energy of the waves is basically confined to the wave-guide.

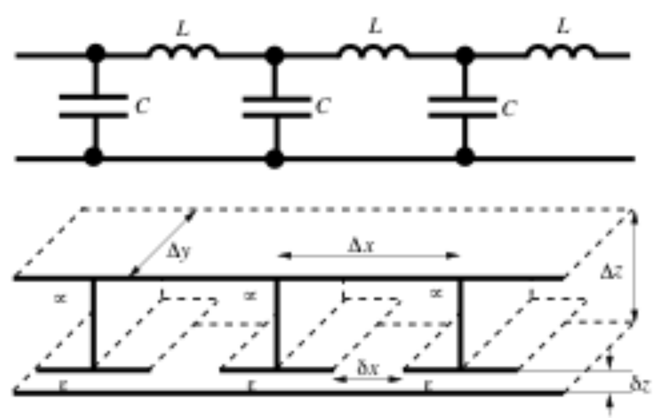

FIG. 1. Circuit diagram and sketch of wave-guide.

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Zitterbewegung and its effects on electrons in semiconductors

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An analogy between the band structure of narrow gap semiconductors and the Dirac equation for relativistic electrons in vacuum is used to demonstrate that semiconductor electrons experience a Zitterbewegung (trembling motion). Its frequency is $\omega_Z \approx \mathcal{E}_g/\hbar$ and its amplitude is λ_Z, where $\lambda_Z = \hbar/m_0^* u$ corresponds to the Compton wavelength in vacuum ($\mathcal{E}_g$ is the energy gap, m_0^* is the effective mass and $u \approx 1.3 \times 10^8$ cm/sec). Once the electrons are described by a two-component spinor for a specific energy band there is no Zitterbewegung but the electrons should be treated as extended objects of size λ_Z. Possible consequences of the above predictions are indicated.

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It was noted in the past that the $E(\mathbf{k})$ relation between the energy E and the wavenumber $\mathbf{k}$ for electrons in narrow-gap semiconductors (NGS) is analogous to that for relativistic electrons in vacuum [1-4]. The analogy was also shown to hold for the presence of external fields which was experimentally confirmed on InSb [5]. This "semirelativity in semiconductors" is valid for time dependent phenomena as well, so that the cyclotron resonance of conduction electrons in InSb could be interpreted in terms of the time dilatation between a moving electron and an observer [5]. The semirelativistic phenomena appear at electron velocities of $10^7 - 10^8$ cm/sec, much lower than the light velocity c. The reason for this is that the maximum velocity u in semiconductors, which plays the role of c in vacuum, is about 10^8 cm/sec.

Until present the semirelativistic considerations for semiconductors were concerned with phenomena related mostly to classical mechanics. The purpose of this contribution is to investigate the quantum domain described by the Hamiltonian for energy bands in NGS, which bears close similarity to the Hamiltonian for relativistic electrons in vacuum. The effects we predict should be much more readily observable in NGS than in vacuum so this taking the limit of large p_i, or by using the analogy: $c = (2m_0c^2/2m_0)^{1/2} \rightarrow (\mathcal{E}_g/2m_0^*)^{1/2} = u$. Taking the experimental parameters $\mathcal{E}_g$ and m_0^* we calculate very similar value of $u \approx 1.3 \times 10^8$ cm/sec for different III-V compounds.

Now we define an important quantity

$$\lambda_Z = \frac{\hbar}{m_0^* u} \ , \tag{2}$$

which we call the length of Zitterbewegung for reasons given below. Here we note that it corresponds to the Compton wavelength $\lambda_c = \hbar/m_0 c$ for electrons in vacuum. Let us suppose that we confine an electron to the dimensions $\Delta z \approx \lambda_Z/2$. Then the uncertainty of momentum is $\Delta p_z \geq \hbar/\Delta z$ and the resulting uncertainty of energy $\Delta E \approx (\Delta p_z)^2/2m_0^*$ becomes

$$\Delta E \geq 2m_0^* u^2 = \mathcal{E}_g \ . \tag{3}$$

Thus the electron confined to $\Delta z \approx \lambda_Z/2$ has the uncertainty of energy larger than the gap, so that it "does not know" whether it belongs to the conduction or to the